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Unit 4:- Analysis of Latin Square Design

In Randomised Block design whole of the experimental area is divided into relatively homogeneous groups (blocks) and treatments are allotated at random to units within each block i.e. randomisation is subjected to one restriction i.e. within blocks. But in field experiment it may happen that experimental area (field) exhibits fertility in strips e.g :- Cultivation may result in alternative strips of high or low fertility. RBD will be effective if the blocks are consituted parallel to these strips and will be extremely inefficient, if the blocks are across the strips. In most of the cases fertility gradient is rarely known. A useful method of eliminating fertility variations consists in an experimental layout which control variation in two perpendicular direction. Such a layout is known as Latin Square Design.

In this design the no. of treatments is equal to the no. of replication. Thus in case of 'm' treatments the no. of experimental units required is $m \times m = m^2$. The whole of the experimental area is divided into m^2 experimental units (plots) arranged in a plot so that each row as well as colomn contain 'm' unit plot.

The 'm' treatments are then allocated at random to those rows and columns in such a may that every treatment occurs once and only once in each row and column.

Such a layout is known as Maximum LSD

For e.g :- 5X5 LSD is

A	B	C	D	E
B	C	D	E	A
C	D	E	A	B
D	E	A	B	C
E	A	B	C	D

"A daily life example can be simple game called Sudoku puzzle is also a special case of Latin Square design."

Statistical analysis of Maximum for one obs^n per experimental Unit:-

Let y_{ijk} $(i, j, k = 1, 2, \ldots, m)$ denotes the response from the unit (plot in the experiment) in the ith row, j^{th} column and receiving the kth treatment the triplet (i, j, k) assumes only m^2 of the possible m^3 values of an Latin square selected by the experiment. If S represents the set of m^2 values then symbolically $(i, j, k) \in S$. If a single observation is made per experimental units then the linear model is

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + e_{ij}, \quad (i, j, k) \in S \quad \text{--- (1)}$$

where μ is the constant mean effect, α_i, β_j and γ_k are the effects due to i^{th} row, j^{th} column and k^{th} treatment respectively and e_{ijk} is the error effect due to random component assumed to be normally distributed with mean zero and variance σ^2 i.e $e_{ijk} \sim N(0, \sigma^2)$

Since α_i, β_j and γ_k are the effects of ith row, j^{th} row and k^{th} treatment.

$$\therefore \sum_i \alpha_i = \sum_j \beta_j \text{ and } \sum_k \gamma_k = 0$$

The least square estimates of u, α_i, β_j and γ_k are obtained by minimising the error sum of square.

Since 'α_i' occurs m times for each 'i', then we have

$$\sum_{i,j,k} \alpha_i = m \sum_{i,j,k} \alpha_i = 0$$

$$\text{Illy } \sum_{i,j,k} \beta_j = m \sum_j \beta_j = 0 \text{ and } \sum_{i,j,k} \gamma_k = m \sum_k \gamma_k = 0$$

$$S = \sum_{i,j,k} e_{ijk}^2$$

$$S = \sum_{(i,j,k \in S)} (y_{ijk} - u - \alpha_i - \beta_j - \gamma_k)^2$$

$$\therefore \frac{\partial S}{\partial u} = 0$$

$$\Rightarrow 2 \sum_{i,j,k} (y_{ijk} - \hat{u} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\gamma}_k) = 0$$

$$\Rightarrow \sum_{ijk} y_{ijk} - \sum_{ijk} \hat{u} - \sum_{ijk} \hat{\alpha}_i - \sum_{ijk} \hat{\beta}_j - \sum_{ijk} \hat{\gamma}_k = 0$$

$$\Rightarrow \sum_{i,j,k} y_{ijk} - m^2 \hat{u} - m \sum_i \hat{\alpha}_i - m \sum_j \hat{\beta}_j - m \sum_k \hat{\gamma}_k = 0$$

$$\Rightarrow \sum_{i,j,k} y_{ijk} - N\hat{u} - 0 - 0 - 0 = 0$$

$$\Rightarrow N\hat{u} = \sum_{i,j,k} y_{ijk} \qquad \left\{ N = m^2 ;\ m \sum \hat{\alpha}_i = m \sum \hat{\beta}_j = m \sum \hat{\gamma}_k = 0 \right\}$$

$$\hat{u} = \frac{1}{N} \sum_{ijk} y_{ijk}$$

$$\boxed{\hat{u} = \bar{y}_{...}}$$

"The main purpose of latin square design is to remove the variability that arises from two factors"

and $\frac{\partial S}{\partial \alpha_i} = 0$

$$\Rightarrow -2\sum_{jk}(y_{ijk} - \hat{\mu} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\gamma}_k) = 0$$

$$\Rightarrow \sum_{jk} y_{ijk} - \sum_{jk}\hat{\mu} - \sum_{jk}\hat{\alpha}_i - \sum_{jk}\hat{\beta}_j - \sum_{jk}\hat{\gamma}_k = 0$$

$$\Rightarrow \sum_{jk} y_{ijk} - m\hat{\mu} - m\hat{\alpha}_i - 0 - 0 = 0$$

$$\Rightarrow \sum_{jk} y_{ijk} - m\hat{\mu} - m\hat{\alpha}_i = 0$$

$$\Rightarrow \hat{\alpha}_i = \frac{1}{m}\sum_{jk} y_{ijk} - \hat{\mu}$$

$$\hat{\alpha}_i = \frac{1}{m}\sum_{jk} y_{ijk} - \bar{y}_{...}$$

$$\boxed{\hat{\alpha}_i = \bar{y}_{i..} - \bar{y}_{...}}$$

alow $\frac{\partial S}{\partial \beta_j} = 0$

$$\Rightarrow -2\sum_{ik}(y_{ijk} - \hat{\mu} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\gamma}_k) = 0$$

$$\Rightarrow \sum_{ik} y_{ijk} - \sum_{ik}\hat{\mu} - \sum_{ik}\hat{\alpha}_i - \sum_{ik}\hat{\beta}_j - \sum_{ik}\hat{\gamma}_k = 0$$

$$\Rightarrow \sum_{ik} y_{ijk} - m\hat{\mu} - m\hat{\beta}_j = 0$$

$$m\hat{\beta}_j = \sum_{ik} y_{ijk} - m\hat{\mu}$$

$$\hat{\beta}_j = \frac{1}{m}\sum_{ik} y_{ijk} - \hat{\mu}$$

$$\boxed{\hat{\beta}_j = \bar{y}_{.j.} - \bar{y}_{...}}$$

also $\frac{\partial S}{\partial \hat{\gamma}_k} = 0$

$$\Rightarrow -2\sum_{ij}(y_{ijk} - \hat{u} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\gamma}_k) = 0$$

$$\Rightarrow \sum_{ij} y_{ijk} - \sum_{ij}\hat{u} - \sum_{ij}\hat{\alpha}_i - \sum_{ij}\hat{\beta}_j - \sum_{ij}\hat{\gamma}_k = 0$$

$$\Rightarrow \sum_{ij} y_{ijk} - \sum_{ij}\hat{u} - \sum_{ij}\hat{\gamma}_k = 0$$

$$\Rightarrow \sum_{ij}\hat{y}_{ijk} - m\hat{u} - m\hat{\gamma}_k = 0$$

$$\Rightarrow \hat{\gamma}_k = \frac{1}{m}\sum_{ij} y_{ijk} - \hat{u}$$

$$\Rightarrow \boxed{\hat{\gamma}_k = \bar{y}_{..k} - \bar{y}_{...}}$$

$$e_{ijk} = y_{ijk} - \hat{u} - \hat{\alpha}_i - \hat{\beta}_j - \hat{\gamma}_k$$

$$e_{ijk} = y_{ijk} - \bar{y}_{...} - \bar{y}_{i..} + \bar{y}_{...} - \bar{y}_{.j.} + \bar{y}_{...} - \bar{y}_{..k} + \bar{y}_{...}$$

$$\boxed{e_{ijk} = y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...}}$$

$\therefore$ Equation ① becomes

$$y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + e_{ij}$$
$$= \bar{y}_{...} + (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{..k} - \bar{y}_{...}) + (y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})$$

$$(y_{ijk} - \bar{y}_{...}) = (\bar{y}_{i..} - \bar{y}_{...}) + (\bar{y}_{.j.} - \bar{y}_{...}) + (\bar{y}_{..k} - \bar{y}_{...}) + (y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})$$

Squaring b/s and taking summation over all the values of i, j & k, we have.

$$\sum_i\sum_j\sum_k (y_{ijk} - \bar{y}_{...})^2 = \sum_{ijk}(\bar{y}_{i..} - \bar{y}_{...})^2 + \sum_{ijk}(\bar{y}_{.j.} - \bar{y}_{...})^2 + \sum_{ijk}(\bar{y}_{..k} - \bar{y}_{...})^2$$
$$+ \sum_{ijk}(y_{ijk} - \bar{y}_{i..} - \bar{y}_{.j.} - \bar{y}_{..k} + 2\bar{y}_{...})^2$$

$\Rightarrow TSS = SSR + SSC + SST + SSE$

where $TSS = \sum_{IJK} (y_{ijk} - \bar{y}_{...})^2$ is the total sum of square and will carry (m^2-1) d.f.

where $SSR = m\sum_{IJK} (\bar{y}_{i..} - \bar{y}_{...})^2$ is the sum of square due to row and will carry $(m-1)$ d.f.

$SSC = m\sum_{J} (\bar{y}_{.j.} - \bar{y}_{...})^2$ is the sum of square due to column and will carry $(m-1)$ d.f.

$SST = m\sum_{K} (\bar{y}_{..k} - \bar{y}_{...})^2$ is the sum of square due to treatment and will carry $(m-1)$ d.f.

where $SSE = TSS - SSR - SSC - SST$ is the sum of square due to error and will carry $(m-1)(m-2)$ d.f.

ANOVA TABLE

Source of Variation	d.f	S.S	M.S.S	Variance Ratio
Row	$(m-1)$	SSR	$S_R^2 = \frac{SSR}{m-1}$	$f_R = \frac{S_R^2}{S_E^2} \sim f^{(\alpha)}_{(m-1,(m-1)(m-2))}$
Column	$(m-1)$	SSC	$S_C^2 = \frac{SSC}{m-1}$	$f_C = \frac{S_C^2}{S_E^2} \sim$ "
Treatment	$(m-1)$	SST	$S_T^2 = \frac{SST}{m-1}$	$f_T = \frac{S_T^2}{S_E^2} \sim$ "
Error	$(m-1)(m-2)$	SSE	$S_E^2 = \frac{SSE}{(m-1)(m-2)}$	
Total	(m^2-1)	TSS		

for Row effect

$H_\alpha : \alpha_1 = \alpha_2 = \cdots = \alpha_m = 0$

for Column effect

$H_\beta : \beta_1 = \beta_2 = \cdots = \beta_m = 0$

for Treatment effect

$H_\gamma : \gamma_1 = \gamma_2 = \cdots = \gamma_m = 0$

If $Calf_R > Tabf_{(m-1);(m-1)(m-2)}$ then we reject H_α at $\alpha\%$ l of s otherwise we may accept it.
Ly if $Calf_C > Tabf_{(m-1)(m-1)(m-2)}$ then we reject H_β at $\alpha\%$ l of s otherwise we may accept it.
And if $calf_T > Tab.f_{(m-1),(m-1)(m-2)}$ then we reject H_γ at $\alpha\%$ l of s otherwise we may accept it.

Advantages of Latin Square Design:-

1. By means of grouping the units in 2 ways, the error variations is reduced by eliminating the two measure source of variation that are not relevent to the comparison between different treatment. It means latin square design is more efficient than R.B.D and C.R.D.

2. Being an incomplete 3-way design, it is economic over the corresponding complete 3 way design. Here instead of m^3 units we shall need only m^2 units.

3. The analysis remain simple with little complication in case of missing data.

4. More than one factor can be investigated simultaneously and with fewer trials than more complicated design.

Disadvantages of Latin Square design:-

1. The fundamental assumption that there is no interaction between different factors. i.e. the factor act independtly may be true in general.

(2) Unlike R.B.D, in LSD the no. of treatments is restricted to the no. of replication and this limits its field of application.

(3) LSD is suitable for the no. of treatment b/w 5 and 10 and for more than 10 and 12 treatments the design is seldom used since in that case the square becomes too large and does not remain homogeneous

(4) In the field layout, RBD is much easy to manage than a square or a rectangular field or a field of any shape where as for the later approximately a square field is necessary.

Factorial Experiment :-

In the foregoing experiments performed either in C.R.D or R.B.D or L.S.D, we were primarily concerned with the comparison and estimation of the effect of a single set of treatments like varities of wheat, manure or different methods of cultivation etc. such experiment which deals with one factor only may be called simple experiment.

In factorial experiment, the effect of several factors of variation are studied and investigated simultaneously, the treatments being all the combination of different factors under study or the experiment in which the effect of a no. of levels of one factors simultaneously are called factorial Experiment.

If we have n factors each at 's' levels then the experiment is called s^n factorial experiment.
for eg:- 2^3 mean 3 factors each at 2 levels.

Advantages :-

1. The most important advantages of factorial experiment is that they help us to study interaction effect or combined effect.
2. It is economic from time level and material point of view.
3. When the experiments are conducted factor by factor much more resources are required for same procedure when they are tried in F.E.
4. The factorial experiments are more effictent.

SOME IMPORTANT DEFINIDTION :-

1. Main Effect :- Main effect of a factor is defined as a measure of change in response due to change in the level of the factor, averaged over all the level of other factors in a factorial experiments.

2. The Interaction between two factors may be defined as the failure of a factor to give the same response at various levels of the other factors.

In general, an interaction is an additional effect due to the combined effect of two or more factors.

(i) Contrast :- A linear combination of $\sum_{i=1}^{k} a_i t_i$ of k treatment means t_i, $i = 1, 2, 3, \ldots, k$ is called contrast if $\sum_{i=1}^{k} a_i = 0$

(ii) Orthogonal Contrast :- The contrast $\sum_{i=1}^{k} a_i t_i$ and $\sum_{i=1}^{k} b_i t_i$ are called Orthogonal contrast if $\sum_{i=1}^{k} a_i b_i = 0$

2^2 factorial Experiment :- In 2^2- factorial exp. there are two factors A and B each at two levels. Let the level of factor A and B is denoted by a_0, a_1 and b_0, b_1 respectively. Then the treatment are the combination of level of two factors A and B i.e. the four treatment combination can be enumerated as follows:-

$a_0 b_0$ or 1 : Factor A and B both at first level

$a_1 b_0$ or a : Factor A at second level and B at first level.

$a_0 b_1$ or b : Factor A at first level and B at second level.

$a_1 b_1$ or 'ab' : A and B are both at second level.

These four combinations can be compared by laying out the experiment in

(i) RBD with 'r' replicates each replicated containing four units.

(ii) 4x4 LSD and ANOVA can be carried out accordingly. In the above cases there are 3 d.f associated with the treatment effect.

In F.E our main objective is to carry

out separate lists for the main effects A, B and the interaction AB splitting the treatment S.S with 3 d.f. into three orthogonal components each with 1 d.f and each associated either with the main effects A and B or the interaction AB.

Main Effect and the Interaction in 2^n factorial Experiment:-

Let [1], [a], [b] and [ab] denotes the total yields of the r- units receiving the treatments 1, a, b and ab respectively and let the corresponding mean values obtained on dividing these totals by r be denoted by [1], [a], [b] and [ab] respectively. The letters A, B and AB when they refer to number will represent the main effect due to the factor A and B and their interaction AB respectively.

The effect of factor A can be represented by the difference between mean yields obtained at each level.

Thus the effect of factor A at the first level b of B the effect of changing 'A' from lower level to upper level in presence of lower level of factor B.

$$= (a_1, b_0) - (a_0 b_0) \quad \text{—— (i)}$$
$$= (a) - (1)$$

The effect of changing A from lower level to upper level in presence of upper level of factor B.

$$= (a_1 b_1) - (a_0 b_1)$$
$$= (ab) - (b) \quad \text{—— (2)}$$

These two effects in ① and ② are termed as simple effect of factor A.

The average observed effect of A over the two levels B is called the main effect due to A is defined by

$$A = \frac{1}{2}\left[(ab)-(b)+(a)-1\right]$$

A simplified form is

$$A = \frac{1}{2}\left[(a-1)(b+1)\right]$$

Ily the main effect of B is $\frac{1}{2}\left[(a+1)(b-1)\right]$ where the R.H.S is to expanded algebrically and the treatment combinations are to replaced by their means.

And $AB = \frac{1}{2}\left[(a-1)(b-1)\right]$

Analysis of 2^2-factorial Experiment:-

Since the factorial experiment are conducted either in C.R.D, R.B.D or L.S.D and can be analysed in the usual manner except that in 2^2-factorial experiment. The treatment sum of square is split into three orthogonal components and each having one d.f. The sum of squares due to any factorial effect A, B and AB can be obtained by multiplying the squares of factorial effect total by a suitable quantity.

The factorial effect total are given by

$$[A] = [ab]-[b]+[a]-1$$

$$[B] = [ab]+[a]-[b]-1$$

$$[AB] = [ab]-[a]-[b]+1$$

These factorial effect total are easily obtained by using sign and divisor method i.e.

Factorial effect	Treatment Means				Divisors
M	(1)	(a)	(b)	(ab)	
1	+	+	+	+	4
a	−	+	−	+	2
b	−	−	+	+	2
ab	+	−	−	+	2

The sum of square due to main effect $A = \frac{[A]^2}{4r}$

The sum of squares due to main effect $B = \frac{[B]^2}{4r}$

The sum of squares due to combined effect $AB = \frac{[AB]^2}{4r}$

where sum of squares due to any factorial effect is obtained by multiplying the square of factorial effect total by $\frac{1}{4r}$. If the expt. is RBD with 'r' replicates then the ANOVA is as follows:-

Source of Variation	d.f	S.S	M.S.S	F (Variance Ratio
Blocks	$r-1$	S_R^2	S_R^2	$F_R = \frac{S_R^2}{S_E^2} \sim F^{(\alpha)}_{(r-1),(3(r-1))}$
Main effect				
A	1	$S_A^2 = \frac{[A]^2}{4r}$	S_A^2	$F_A = \frac{S_A^2}{S_E^2} \sim F^{(\alpha)}_{(1,3(r-1))}$
B	1	$S_B^2 = \frac{[B]^2}{4r}$	S_B^2	$F_B = \frac{S_B^2}{S_E^2} \sim F^{(\alpha)}_{(1,3(r-1))}$
Interaction				
AB	1	$S_{AB}^2 = \frac{[AB]^2}{4r}$	S_{AB}^2	$F_{AB} = \frac{S_{AB}^2}{S_E^2} \sim$ "
Error	$3(r-1)$	S_E^2	S_E^2	
Total	$4(r-1)$			

Here each of the statistic F_A, F_B, F_{AB} follows F-distⁿ with 1 and $3(n-1)$ d.f.
If for any factorial experiment, calculated F is greater than Tabulated F at $[1, 3(n-1)]$ d.f at certain l.o.f.s then the hypothesis of presence of factorial effect is rejected otherwise it may be accepted.

YATES PROCEDURE FOR CALCULATING THE FACTORIAL EFFECT TOTAL FOR FACTORS:-

Treatment combination	Total Yield			Effect total
	[1]	[a] + [1]	[ab] + [b] + [a] + [1]	G
a	[a]	[ab] + [b]	[ab] − [b] + [a] − [1]	[A]
b	[b]	[a] − [1]	[ab] + [b] − [a] − [1]	[B]
ab	[ab]	[ab] − [b]	[ab] − [b] − [a] + [1]	[AB]

2^3 Factorial Experiment:-

In 2^3 factorial experiment we have three factors A, B and C each at two levels say (a_0, a_1) (b_0, b_1) (c_0, c_1) then the eight treatment in standard systematic order are 1, a, b, ab, c, ac, bc, abc. These 8 treatment can be compared either in C.R.D, R.B.D or L.S.D. Here treatment sum of square will carry 7 d.f. and is split into seven mutually orthogonal components each having one d.f.

Main Effects And Interaction:-

Let [1], [a], [b], [c], [ab], [bc], [abc] be the totals from the corresponding treatments from r-replications and (1), (a), (b), (c), (ab), (bc), (abc) be their corresponding means. i.e. $(1) = \frac{[1]}{r}$

$$(a) = \frac{[a]}{r} \quad \text{------} \quad (abc) = \frac{[abc]}{r}$$

Then the four simple effects of A which can be obtained as

Level of B	Level of C	Simple Effect of A
b_0	c_0	$(a_1b_0c_0) - (a_0b_0c_0) = (a) - (1)$
b_1	c_0	$(a_1b_1c_0) - (a_0b_1c_0) = (ab) - (b)$
b_0	c_1	$(a_1b_0c_1) - (a_0b_0c_1) = (ac) - (c)$
b_1	c_1	$(a_1b_1c_1) - (a_0b_1c_1) = (abc) - (bc)$

The main effect of A is the average of these four simple effect of A.

$$\text{Main effects of } A = \frac{1}{4}\left[(abc) - (bc) + (ac) - (c) + (ab) - (b) + (a) - 1\right]$$

$$= \frac{1}{4}\left[(a-1)(b+1)(c+1)\right]$$

$$\text{Illy Main effect of } B = \frac{1}{4}\left[(a+1)(b-1)(c+1)\right]$$

$$\text{and Main effect of } C = \frac{1}{4}\left[(a+1)(b+1)(c-1)\right]$$

where the sign in each parathesis is -ve if the corresponding factor is included in the effect otherwise it is +ve.

(i) The combined effect of interaction AB at first or lower level of C is the average differences in the effect of factor A at two level of factor B.

$$[AB]_1 = \frac{1}{2}\left[(ab)-(b)-(a)+(1)\right]$$

(ii) The effect of interaction AB at second level of factor C is the average difference in the effect of factor A at the level of factor B.

$$[AB]_2 = \frac{1}{2}\left[(abc)-(bc)-(ac)+(c)\right]$$

The interaction AB is defined as the average of these two combined or interaction effects.

$$\text{Interaction effect } AB = \frac{1}{2}\left[[AB]_1 + [AB]_2\right]$$

$$= \frac{1}{4}\left\{(abc)-(bc)-(ac)+(c)+(ab)-(b)-(a)+(1)\right]$$

$$= \frac{1}{4}\left[(a-1)(b-1)(c+1)\right]$$

Illy, Interaction effect $BC = \frac{1}{4}\left[(a+1)(b-1)(c-1)\right]$

and Interaction effect $AC = \frac{1}{4}\left[(a-1)(b+1)(c-1)\right]$

It is to be noted that $AB = BA$, $AC = CA$ and $BC = CB$.

The interaction effect ABC is the average differences of $[AB]_2$ and $[AB]_1$.

$$\therefore \text{The interaction effect } ABC = \frac{1}{4}\left[(abc)-(bc)-(ac)-(ab)+(c)+(a)+(b)-1\right]$$

$$= \frac{1}{4}\left[(a-1)(b-1)(c-1)\right]$$

If M denotes the mean yied from all 'n' replicates then.

$$M = \frac{1}{8}\left[(a+1)(b+1)(c+1)\right]$$

Analysis of 2^3-factorial Experiment:-

Factorial Effect	(1)	(a)	(b)	(ab)	(c)	(ac)	(bc)	(abc)	Divisor
M	+	+	+	+	+	+	+	+	8
A	−	+	−	+	−	+	−	+	4
B	−	−	+	+	−	−	+	+	4
C	−	−	−	−	+	+	+	+	4
AB	+	−	−	+	+	−	−	+	4
AC	+	−	+	−	−	+	−	+	4
BC	+	+	−	−	−	−	+	+	4
ABC	−	+	+	−	+	−	−	+	4

By using the table of signs and divisor of a 2^3 factorial experiment the various factorial effect totals can be expressed as mutually orthogonal contrast of 8 treatment totals.

$$[A] = [abc] - [bc] - [ac] - [c] + [ab] - [b] + [a] - [1]$$
$$[B] = [abc] + [bc] - [ac] - [c] + [ab] + [b] - [a] - [1]$$
$$[C] = [abc] + [bc] + [ac] + [c] - [ab] - [b] - [a] - [1]$$

Illy $[ABC] = [abc] - [bc] - [ac] + [c] - [ab] + [b] + [a] - [1]$

Then the treatment sum of squares with 7 d.f is split up into 7 mutually orthogonal contrast each with one d.f. the sum of squares due to any factorial effect can be obtained by multiplying the squares of factorial effect total by $\frac{1}{2^3 r}$ or $\frac{1}{8r}$.

Sum of square due to main effect $[A] = \frac{[A]^2}{8r}$

" $[B] = \frac{[B]^2}{8r}$

" $[C] = \frac{[C]^2}{8r}$

" $[AB] = \frac{[AB]^2}{8r}$

" $[BC] = \frac{[BC]^2}{8r}$

" $[AC] = \frac{[AC]^2}{8r}$

" $[ABC] = \frac{[ABC]^2}{8r}$

If the experiment is R.B.D with r replicates then the Anova Table for 2^3 factorial experiment is on next page.

ANOVA TABLE

Source of Variation	d.f	S.S	M.SS	F-Ratio
Blocks	$(r-1)$	SSB	S^2_B	$F_B = \frac{S^2_B}{S^2_E} \sim F^d_{(r-1,\,7(r-1))}$
Treatment	7	SST	S^2_T	
Main Effect				
A	1	S^2_A	S^2_A	$F_T = \frac{S^2_T}{S^2_E} \sim F^d_{(7,\,7(r-1))}$
B	1	S^2_B	S^2_B	
C	1	S^2_C	S^2_C	
Interaction				
AB	1	S^2_{AB}	S^2_{AB}	$F_A = \frac{S^2_A}{S^2_E} \sim F^d_{(1,\,7(r-1))}$
BC	1	S^2_{BC}	S^2_{BC}	F_B = "
AC	1	S^2_{AC}	S^2_{AC}	F_C = "
Interaction				
ABC	1	S^2_{ABC}	S^2_{ABC}	
Error	$7(r-1)$	SSE	S^2_E	
Total	$8(r-1)$			

Here each of the statistics $F_A, F_B, \ldots, F_{ABC}$ follows some dist^n if $F_{cal.}$ is greater than $F_{tab.}$ for any factorial effect at certain level of significance 'α' then we reject H_0 of presence of that factorial effect otherwise we may accept H_0.

YATES PROCEDURE FOR CALCULATING THE FACTORIAL EFFECT FOR THREE FACTORS :-

Treatment Combination	Total Yield				S.S
1	[1]	$[a]+[1]=U_1$	$U_2+U_1=V_1$	$V_2+V_1=[1]$	$G^2/32$
a	[a]	$[ab]+[b]=U_2$	$U_4+U_3=V_2$	$V_4+V_3=[A]$	$A^2/32$
b	[b]	$[ac]+[c]=U_3$	$U_6+U_5=V_3$	$V_6+V_5=[B]$	$B^2/32$
ab	[ab]	$[abc]+[bc]=U_4$	$U_8+U_7=V_4$	$V_8+V_7=[AB]$	$AB/32$
c	[c]	$[a]-[1]=U_5$	$U_2-U_1=V_5$	$V_2-V_1=[C]$	$C/32$
ac	[ac]	$[ab]-[b]=U_6$	$U_4-U_3=V_6$	$V_4-V_3=[AC]$	$AC/32$
bc	[bc]	$[ac]-[c]=U_7$	$U_6-U_5=V_7$	$V_6-V_5=[BC]$	$BC/32$
abc	[abc]	$[abc]-[bc]=U_8$	$U_8-U_7=V_8$	$V_8-V_7=[ABC]$	$ABC/32$